Please do the following ten problems. Write your UID number ONLY, not your name.

(1.) Let \( L : C \to D \) be a functor, left adjoint to \( R : D \to C \). Show: if the counit \( L \circ R \to id_D \) is a natural isomorphism, then \( R \) is fully faithful.

(2.) Let \( A \) be a central division algebra (of finite dimension) over a field \( k \). Let \( [A, A] \) be the \( k \)-subspace of \( A \) spanned by the elements \( ab - ba \) with \( a, b \in A \). Show that \( [A, A] \neq A \).

(3.) Given \( \phi : A \to B \) a surjective morphism of rings, show that the image by \( \phi \) of the Jacobson radical of \( A \) is contained in the Jacobson radical of \( B \).

(4.) Let \( G \) be a group and \( H \) a normal subgroup of \( G \). Let \( k \) be a field and let \( V \) be an irreducible representation of \( G \) over \( k \). Show that the restriction of \( V \) to \( H \) is semisimple.

(5.) Let \( G \) be a finite group acting transitively on a finite set \( X \). Let \( x \in X \) and let \( P \) be a Sylow \( p \)-subgroup of the stabilizer of \( x \) in \( G \). Show that \( N_G(P) \) acts transitively on \( X^P \).

(6.) Let \( A \) be a ring and \( M \) a noetherian \( A \)-module. Show that any surjective morphism of \( A \)-modules \( M \to M \) is an isomorphism.

(7.) Let \( G \) be a finite group and let \( s, t \in G \) be two distinct elements of order 2. Show that the subgroup of \( G \) generated by \( s \) and \( t \) is a dihedral group. (Recall that the dihedral groups are the groups \( D(m) = \langle g, h \mid g^2 = h^2 = (gh)^m = 1 \rangle \) for some \( m \geq 2 \)).

(8.) Let \( F \) be a finite field. Without using any of the theorems on finite fields, show that \( F \) has a field extension of degree 2.

(9.) Let \( G \) be a finite group. Show that there exist fields \( F \subset E \) such that \( E/F \) is Galois with group \( G \).

(10.) Let \( F \) be a field. Show that the polynomial ring \( F[t] \) has infinitely many prime ideals. Also prove that algebraically closed fields are infinite.